NITheP cordially invites you to a seminar by:

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Date: Friday, 25th July 2014
Time: 12h20 – 13h20
Venue: NITheP Seminar Room, H-Block, 3rd Floor

TITLE: Stability switches in singularly perturbed dynamical systems

ABSTRACT:
In recent years a demand for more accurate description of real life processes and advances in experimental techniques have resulted in construction of very complex mathematical models, consisting of large systems of highly coupled nonlinear differential equations. The sheer size and complexity of such models often precludes any robust, be it theoretical or numerical, analysis of them. For- tune- tunately, often such models describe phenomena occurring on vastly different time or size scales. Such a structure offers hope that it can be simplified by focusing at one scale relevant to the problem at hand and averaging over the other scales, without affecting too much the salient features of the original dynamics. The existence of, say, different time scales in a model is often reflected by a scale parameter given by the ratio of typical scales of different process driving the model. Then the model operates close to the slow or the fast regime if the scale parameter is, respectively, small or large. However, most often simply replacing in the model the relevant parameter by its critical value results in a dramatic change of the properties of the model, rendering this approach useless. In such a case to understand the behaviour of the model close to the boundaries between different regimes one has to carefully analyse the limit of the solutions as the scale parameter tends to the critical value and to construct an approximation which would describe this limit. We will focus on complex processes described by systems of ordinary differential equations. In such cases the ones with two time scales often can be written in the so-called Tikhonov form

\[
\begin{align*}
x' &= f(x_\varepsilon, y_\varepsilon, t, \varepsilon) \\
\varepsilon y' &= g(x_\varepsilon, y_\varepsilon, t, \varepsilon),
\end{align*}
\]

with \((x_\varepsilon, y_\varepsilon) \in \mathbb{R}_{n\times m}\). Tikhonov theorem gives a set of conditions under which \((x_\varepsilon, y_\varepsilon) \rightarrow (x, y)\), where \((x, y)\) is the solution of the limit equation obtained by setting \(\varepsilon = 0\). Apart from some technical conditions one of the most important ones is that the degenerated equation \(0 = g(x, y, t, 0)\) has an isolated solution, called quasi steady state, which is attractive. In many cases this assumption is not satisfied and we have quasi steady states which intersect with each other and switch attractiveness at the point of intersection. In such cases, one would expect that, as \(\varepsilon\) tend to zero, the solutions tend to the attractive branch of the closest steady state and switch to the other attractive branch after passing through the intersection point. However, in many cases the system behaves in a different way, namely, the solution follows the repelling branch of the quasi steady state and only after some fixed time it suddenly jumps to the other attractive branch of the quasi steady state.

In the talk we shall present some models of this type and present mathematical tools for their analysis.