QCD AND HADRONS AT FINITE TEMPERATURE:
PROBING QUARK-GLUON DECONFINEMENT

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QUARK-GLUON DECONFINEMENT
PHASE DIAGRAM: T, \( \mu \)
TWO PHASES: QCD & HADRONIC

PURELY NUMERICAL

LATTEICE QCD: FROM QCD SECTOR

ANALYTICAL: $T \to T_C$

HOT (PERTURBATIVE) QCD: $T >> T_C$

QCD SUM RULES (BLEND: QCD & HADRONIC): $T = 0 \to T_C$
T = 0
COMPLEX SQUARED ENERGY \((s = E^2)\) PLANE

BOUND STATES \((\tau = \infty)\)

RESONANCES \((\tau < \infty)\)
QUANTUM MECHANICS

\[ E = 0 \]

\[ E < 0 \]
The diagrams illustrate the concepts of different Riemann sheets. The top diagram shows a point marked on the complex plane, representing a specific location in the first Riemann sheet. The bottom diagram continues the path through the complex plane, ending at the same point, now within the second Riemann sheet, as indicated by the label "SECOND RIEMANN SHEET."
Realistic Spectral Function

\[ Im \Pi \]

\[ s_0 \equiv E^2 \]
QUANTUM CHROMODYNAMICS

\[ \mathcal{L} = i \bar{\psi}_a (x) \gamma_\mu \partial^\mu \psi_a (x) - m_0 \bar{\psi}_a (x) \psi_a (x) - \frac{1}{4} F_{i \mu \nu} F_{i}^{\mu \nu} \]

\[ - g G_{i \mu} (x) \bar{\psi}_a (x) \gamma^\mu \lambda^{i}_{ab} \psi_b (x) \]

\[ i = 1, 2, \ldots, 8 \quad a, b = 1, 2, N_c = 3 \quad n_F = 6 \]

\[ F_{i \mu \nu} \equiv \partial_\mu G_{i \nu}^{i} - \partial_\nu G_{i \mu}^{i} - g f_{ijk} G_{i \mu}^{j} G_{k \nu}^{k} \]
QCD SYMMETRIES

LOCAL GAUGE SYMMETRY SU(3)_{COLOUR}

FLAVOUR SU(2) & SU(3): m_u = m_d = m_s : \partial^\mu V_\mu \propto \Delta m_q

CHIRAL SU(2)_V \times SU(2)_A & SU(3)_V \times SU(3)_A : m_u = m_d = m_s = 0

REALIZATION

WIGNER – WEYL: CLASSIFICATION: \mathcal{L} \implies \text{states}

NAMBU-GOLDSTONE: \mathcal{L} \not\implies \text{states}
QCD Low Energy Theorem

- $\pi \rightarrow \mu \bar{\nu}_\mu : \langle 0 | A_\alpha (0) | \pi (p) \rangle = i p_\alpha f_\pi$

- $\partial A = f_\pi M_\pi^2 \phi_\pi$ \quad N.G. \quad M_\pi = 0, \quad f_\pi \neq 0$

- Gell-Mann – Oakes – Renner

- $- (m_u + m_d) \langle 0 | \bar{u} u + \bar{d} d | 0 \rangle = 2 f_\pi^2 M_\pi^2$

- $f^2 \propto |\langle 0 | \bar{\alpha} \alpha | 0 \rangle| \quad \& \quad M^2 \propto m$
CONFINEMENT

• STRONG MODIFICATION TO QUARK & GLUON PROPAGATORS NEAR THE MASS SHELL

• INCORPORATE CONFINEMENT THROUGH A PARAMETRIZATION OF PROPAGATOR CORRECTIONS

IN TERMS OF QUARK & GLUON VACUUM CONDENSATES
\[ S_F = \frac{i}{p - m} \quad \Rightarrow \quad \frac{i}{p - m + \sum(p^2)} \]

\[ D_F = \frac{i}{k^2} \quad \Rightarrow \quad \frac{i}{k^2 + \Lambda(p^2)} \]
\[ \Pi_{\mu\nu}(q^2) = i \int d^4x \ e^{i qx} \langle 0 | T [V_\mu(x) \ V_\nu(0)] | 0 \rangle \]
QUARK CONDENSATE

\[ \langle 0 | \bar{q} \, q | 0 \rangle \]
\[ \langle 0 | \alpha_s G^a_{\mu \nu} G^a_{\mu \nu} | 0 \rangle \]
\[ \Pi(q^2) = \int d^4 x e^{i q x} < 0 | T (J(x) J^+(0) | 0 > \]

\[ \Pi(q^2)|_{QCD} = I + \sum_{N=0} C_{2N+2}(q^2, \mu^2) < 0 | \hat{O}_{2N+2}(\mu^2) | 0 > \]

\[ I \Rightarrow O(\alpha_s^4) \quad C_{2N+2} \Rightarrow \frac{1}{(-q)^{2N+2}} \]

\[ m_q < 0 | q \bar{q} | 0 >, \quad < 0 | \alpha_s G_{\mu\nu} G^{\mu\nu} | 0 >, \quad etc. \]
\[ \Pi(q^2) = i \int d^4 x e^{i q x} <0|T(J(x) J^+(0))|0> \]

\[ J(x) \Rightarrow \begin{cases} \text{EXPERIMENT (DATA)} \\ \text{PARAMETRIZATION} \end{cases} \]

\[ M_H \ g_H \ \Gamma_H \ s_0 \]
QUARK-HADRON DUALITY
QUARK-HADRON DUALITY

\[ -\frac{1}{2\pi i} \oint_{|s_0|} ds \Pi(s)_{QCD} = \int_{s_{th}}^{s_0} ds \frac{1}{\pi} \text{Im} \Pi(s)_{HAD} \]
FINITE TEMPERATURE
$T \neq 0$

HADRONIC SECTOR
Realistic Spectral Function

\[ \text{Im} \, \Pi \]

\[ \tilde{s} \equiv E^2 \]
Realistic Spectral Function \((T)\)

\[ \text{Im} \ \Pi \]

\[ s_0(T) \]

\[ s_0(0) \]

\[ s \equiv E^2 \]
RESONANCE BROADENING  
(Dominguez & Loewe 1988)

- WIDTH: $\Gamma(T)$  
- LIFETIME: $\tau = \Gamma^{-1}$

- **HADRONIC INTERPRETATION:**

  ABSORPTION OF PARTICLES IN MEDIUM

- **QCD INTERPRETATION:**
  
  - **QUARK-GLUON DECONFINEMENT**
ORDER PARAMETERS

$S_0(T) \ & WIDTH(T) \ [M(T)]$
DECONFINEMENT SIGNALS

- RESONANCE BROADENING WITH (increasing) T

- ELECTROMAGNETIC & HADRONIC RADII OF HADRONS: SWELLING WITH (increasing) T

- HADRONIC COUPLINGS: VANISHING WITH (increasing) T

- HADRONIC (weak) DECAY CONSTANTS: VANISHING WITH (increasing) T
T \neq 0

QCD SECTOR
ANNIHILATION: $q^2 > 4 \, m^2_q$
SCATTERING: $q^2 < 0$
\[ \text{Im } \Pi(q^2, T) \propto \frac{\omega}{|\vec{q}|^3} F[n_F(T)] \]

\[ -1 \leq \frac{\omega}{|\vec{q}|} \leq +1 \]

\[ \lim_{\omega \to 0, |\vec{q}| \to 0} \left( \frac{\omega}{|\vec{q}|^3} \right) = \frac{2}{3} \delta(\omega^2) \]
\[ S_F(T = 0) = \frac{k + m}{k^2 - m^2 + i\epsilon} \]

\[ S_F(T) = (k + m) \left[ \frac{1}{k^2 - m^2 + i\epsilon} + 2\pi i \delta(k^2 - m^2) n_F(|k_0|) \right] \]

\[ n_F(z) = \frac{1}{e^{z/T} + 1} \]

\[ S_F(T, \mu) = [(p_0 + \mu)\gamma_0 - \vec{p} \cdot \vec{\gamma} + m] \left\{ \frac{i}{(p_0 + \mu)^2 - \vec{p}^2 - m^2 + i\epsilon} \right. \]

\[ \left. - 2\pi \bar{n}_F(p_0 + \mu) \delta [(p_0 + \mu)^2 - \vec{p}^2 - m^2] \right\} \]

\[ \bar{n}_F(p_0 + \mu) = \theta(p_0 + \mu) n_F(p_0) + \theta(-p_0 - \mu) n_F(-p_0) . \]
RESULTS [Input: $<0| \bar{q} \ q|0>(T)$]

1) LIGHT PSEUDOSCALAR CHANNEL: $\pi$-MESON
   $s_0(T)$

2) LIGHT VECTOR CHANNEL: $\rho$-MESON
   $s_0(T), f_\rho(T), M_\rho(T), \Gamma_\rho(T), C_4<\mathcal{O}_4>(T), C_6<\mathcal{O}_6>(T)$

3) LIGHT AXIAL-VECTOR CHANNEL: $\pi$-MESON & $a_1$ – MESON
   $s_0(T), f_a(T), \Gamma_a(T)$

4) HEAVY-LIGHT PSEUDOSCALAR & VECTOR MESONS
   $s_0(T), f(T), M(T), \Gamma(T)$

5) HEAVY (CHARM) SCALAR, PSEUDOSCALAR, VECTOR MESONS
   $s_0(T), f(T), M(T), \Gamma(T)$
$f_{\pi}^2 \propto |\langle 0 | q \bar{q} | 0 \rangle|$
\( S_0 (T) \)
\( \Gamma_{\rho}(T) \)
$f_{\rho}(T)$
\[ \left\langle 0 \right| \alpha_s G_{\mu\nu}^a G_{\mu\nu}^a \left| 0 \right\rangle (T) \]
\[ C_6 \langle 0 | O_6 | 0 \rangle (T) \propto | \langle 0 | \bar{q} q | 0 \rangle |^2 \]
Fig. 2.2 Excess dimuons compared to theoretical predictions [16], renormalized to the data in the mass interval $M < 0.9$ GeV. No acceptance correction applied.
HEAVY-LIGHT QUARK HADRONS

D & D*  B & B*
$S_0(T)$
\( M_p (T) \)
$f_p(T)$
$\Gamma_p (T)$
HEAVY-HEAVY QUARK HADRONS

$J/\psi$, $\eta_c$, $\chi_c$
$S_0(T)$
$f_V(T)$

Graph showing $f_V(T) / f_V(0)$ as a function of $T / T_c$. The graph exhibits a sharp transition near $T / T_c = 1$. The y-axis ranges from 1.0 to 1.4, and the x-axis ranges from 0.0 to 1.2. The curve starts at 1.0 and rises sharply near $T / T_c = 1$.
$\Gamma_V(T)$
ELECTROMAGNETIC RADIUS OF PION
CONVINCING (?) EVIDENCE FOR DECONFINEMENT PHASE TRANSITION AT $T_c \approx 180 - 200$ MeV