A Universal Concept for Particles in Local Quantum Physics

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Overview

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2. The Setting of Algebraic Quantum Field Theory
3. Detectors and Evolution of Physical States
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Wigner Particles

Wigner’s Analysis
Irreducible unitary representations of the Poincaré group (inhomogeneous Lorentz group) can be classified using two parameters $m$ and $s$, where $m$ has continuous values in $\mathbb{R}$ and $s$ can have integer or half-integer values (Wigner 1939).

Physical Interpretation
States of a stable elementary particle belong to an irreducible representation of the Poincaré group with corresponding mass $m$ and spin $s$.

BUT:

Particles carrying an electric charge do NOT fit into this analysis, since the Lorentz symmetry is broken in theories with long-range interactions!
Quantum Electrodynamics

Gauß’ Law

- Charge $q$ of a state can be determined from its electrical flux distribution at spacelike infinity

$$
\Phi(n) = \lim_{r \to \infty} r^2 n \cdot E(r \cdot n).
$$

- This asymptotic flux distribution commutes with every local measurement and is thus a $c$-number function

$$
n \mapsto \varphi(n)
$$

in each superselection sector (irreducible representation of the quasilocal algebra), i.e. it characterises the superselection sector like the charge.
Quantum Electrodynamics (cont.)

**Question**
Can this superselection sector at the same time accommodate an irreducible representation of the Lorentz group?

**Answer: No!**
This is NOT possible! The mathematically rigorous proof of this fact is based on the observation that the existence of a mass operator $M$ in a superselection sector forces the asymptotic field configuration $\varphi(n)$ to vanish (Buchholz 1986).
More Physical Argument

The given flux distribution \( \varphi \) characterising the superselection sector consists of the flux distribution of the charged particle with momentum \( p \), \( \varphi_p \), and the accompanying radiation field of photons, \( \varphi_{\text{rad}} \), such that

\[
\varphi = \varphi_p + \varphi_{\text{rad}}.
\]

The radiation field carries energy so that the mass of the charged particle itself is blurred.
Infraparticle Problem

Infraparticles

Particles that exhibit this phenomenon are called *infraparticles* (modelled after the inevitable clouds of soft photons accompanying charged states as indicated above).

Question

How can mass and spin of a particle be defined in a mathematically consistent way in the case of infraparticles when Wigner’s approach is not available?
The Framework of Local Quantum Field Theory

- Net of local $C^*$-algebras associated with bounded spacetime regions $\mathcal{O}$, $\mathbb{R}^{s+1} \supset \mathcal{O} \mapsto \mathcal{A}(\mathcal{O}) \subseteq \mathcal{B}(\mathcal{H})$, defining the quasilocal algebra $\mathcal{A} = \bigcup_{\mathcal{O}} \mathcal{A}(\mathcal{O})$, such that
  - $\mathcal{O}_1 \subseteq \mathcal{O}_2 \Rightarrow \mathcal{A}(\mathcal{O}_1) \subseteq \mathcal{A}(\mathcal{O}_2)$ (isotony),
  - there exists an automorphic representation $\alpha_{(\Lambda,x)}$ of the Poincaré group $\mathbb{P}^+_{\Lambda}$ such that $\alpha_{(\Lambda,x)}(\mathcal{A}(\mathcal{O})) = \mathcal{A}(\Lambda \mathcal{O} + x)$ (Poincaré covariance),
  - if the bounded regions $\mathcal{O}_1$ and $\mathcal{O}_2$ are spacelike separated, then $[\mathcal{A}(\mathcal{O}_1), \mathcal{A}(\mathcal{O}_2)] = 0$ (Einstein causality, local commutativity),
  - if $\mathcal{O}_2$ belongs to the causal shadow of $\mathcal{O}_1$ then $\mathcal{A}(\mathcal{O}_2) \subseteq \mathcal{A}(\mathcal{O}_1)$ (primitive causality).
The representation of the subgroup of translations of $P_+^{\uparrow}$ is implemented on $\mathcal{A}$ by a strongly continuous unitary group $U(x) = \exp(i P^\mu x_\mu)$, $x \in \mathbb{R}^{s+1}$, with generators $P^\mu$ having their joint spectrum in the forward light cone $\overline{V}_+ \doteq \{ p \in \mathbb{R}^{s+1} \mid p \cdot p = p^\mu p_\mu \geq 0, p^0 \geq 0 \}$ (spectrum condition).

There exists a unique invariant vacuum state $\Omega$.

Physical states are normalised positive functionals $\omega$ on the quasilocal algebra $\mathcal{A}$: $\omega(A) = \text{Tr}(\rho_\omega A)$, $A \in \mathcal{A}$, where $\rho_\omega \in \mathcal{I}$ is a positive trace-class operator (density matrix).
Phase Space Structure

Phase Space Criterion (Fredenhagen-Hertel)

In a local quantum field theory allowing for a particle interpretation the operators

\[ \Pi_{\beta, \mathcal{O}} : \mathcal{T}_\beta \rightarrow \mathcal{A}(\mathcal{O})^* \quad \phi \mapsto \phi \upharpoonright \mathcal{A}(\mathcal{O}) \]

mapping the normal linear functionals with bounded energy into the dual of the local algebras ought to be compact.

- \[ \mathcal{T}_\beta \doteq \{ \phi \in \mathcal{T} : \exp(\beta H)\phi \exp(\beta H) \in \mathcal{T} \} ; \]
- \[ \text{Norm} \quad \| \phi \|_\beta \doteq \| \exp(\beta H)\phi \exp(\beta H) \|. \]
Particles in Spacetime

Particle Concept

- Particles are physical systems that stay singly localised in their temporal evolution.
- They are detected by certain specific measuring devices (particle counters) $C$. 
Asymptotic Particle Structure in Massive Theories

In massive theories with known asymptotic structure one has (Araki and Haag 1967)

$$\lim_{t \to \infty} \langle \Phi | t^3 C(h, t) | \Psi \rangle = \sum' \int d^3 p \Gamma_{ij}(p) \langle \Phi | a_{j\text{out}}^{\dagger}(p) a_{i\text{out}}(p) | \Psi \rangle h(v_i),$$

where $C(h, t)$ stands for a detector approaching timelike infinity with certain velocity (support of function $h$), and

$$\Gamma_{ij}(p) \doteq 8\pi^3 \langle p j | C(0) | p i \rangle,$$

$$v_i \doteq (p^2 + m_i^2)^{-1/2} p.$$
Detectors

Characteristics of detectors according to Araki and Haag

- Localisation of states.
- Insensitive to the vacuum ($C\Omega = 0$).

Reeh-Schlieder Theorem

Strict localisation is in conflict with vacuum annihilation; no strictly local operator can annihilate the vacuum.

Almost Local Operators

An operator $L_0$ in the quasi-local algebra $\mathcal{A}$ is called *almost local* if there exists a net $\{L_r \in \mathcal{A}(\mathcal{O}_r)\}_{r>0}$ of local operators in the standard diamonds $\mathcal{O}_r = \{(x^0, \mathbf{x}) \in \mathbb{R}^{s+1} \mid |x^0| + |\mathbf{x}| < r\}$ such that for any $k \in \mathbb{N}_0$

$$\lim_{{r \to \infty}} r^k \|L_0 - L_r\| = 0.$$
Asymptotic Particle Structure in Arbitrary Theories

We want to develop a decomposition like the one given by Araki and Haag in a *model-independent* framework!

Characteristics of detectors according to Buchholz and Porrmann

- Localisation of states.
- A minimal energy has to be deposited to trigger a detector (more specific than the condition $C\Omega = 0$ used by Araki and Haag).
An operator $L_0 \in \mathcal{A}$ is said to have the \textit{vacuum annihilation property} if the mapping

$$\mathbb{R}^{s+1} \ni x \mapsto \alpha_x(L_0) = U(x)L_0U(x)^* \in \mathcal{A}$$

has a Fourier transform with compact support $\Gamma$ contained in the complement of the closed forward light cone $\overline{V}_+$. For technical reasons we require in addition smoothness of the operator in the sense that the mapping $(\Lambda, x) \mapsto \alpha(\Lambda, x)(L_0)$ is \textit{infinitely often differentiable} on $P^\uparrow_+$ with respect to the norm topology of $\mathcal{A}$. 
Almost local vacuum annihilation operators satisfying the smoothness assumption constitute a *subspace* $\mathcal{L}_0$ of $\mathcal{A}$.

A *left ideal* in $\mathcal{A}$ is introduced by

$$\mathcal{L} \triangleq \mathcal{A}\mathcal{L}_0 = \text{span}\{AL_0 \mid A \in \mathcal{A}, L_0 \in \mathcal{L}_0\}.$$ 

The *algebra of detectors* $\mathcal{C}$ is defined as a (non-closed, non-unital) *-*subalgebra of $\mathcal{A}$ by

$$\mathcal{C} \triangleq \mathcal{L}^*\mathcal{L} = \text{span}\{L_1^*L_2 \mid L_1, L_2 \in \mathcal{L}\}.$$
Example

Let $A \in \mathfrak{A}$ be an almost local operator and let $\mu$ be the Haar measure on $P^\uparrow_+$, then

$$A(F) = \int d\mu(\Lambda, x) \, F(\Lambda, x) \, \alpha(\Lambda, x)(A)$$

defines an almost local vacuum annihilation operator complying with the smoothness assumption, if the infinitely differentiable function $F$ is rapidly decreasing on the subgroup $\mathbb{R}^{s+1}$ and compactly supported on $L^\uparrow_+$, and has the additional property that the Fourier transforms of the partial functions $F_{\Lambda}(\cdot) \doteq F(\Lambda, \cdot)$ have common support in the compact set $\Gamma \subseteq \overline{C V}_+$ for any $\Lambda \in L^\uparrow_+$. 
Physical States of Bounded Energy

Localisation Centres

Heuristically, given a physical state $\omega$ of bounded energy $E$, a counter $C \in \mathcal{C}$, requiring the minimal energy $\delta$ to be triggered, should detect at most $N \leq E/\delta$ localization centres in $\omega$ at time $t$, this means,

$$\int_{\mathbb{R}^s} d^s x \left| \omega(\alpha(t,x)(C)) \right| < \infty.$$ 

Question

Can we establish this as a fact in our model-independent framework?
Proposition

Let \( E(\Delta) \) be the spectral projection corresponding to the bounded Borel set \( \Delta \) and let \( L_0 \in \mathcal{L}_0 \) have energy-momentum transfer \( \Gamma \) in a convex subset of \( \overline{\mathcal{C}V_+} \). Then (for compact \( K \subseteq \mathbb{R}^s \))

\[
\int_K d^s x \ E(\Delta) \alpha_x (L_0^* L_0) E(\Delta) \xrightarrow{K \to \mathbb{R}^s} \int_{\mathbb{R}^s} d^s x \ E(\Delta) \alpha_x (L_0^* L_0) E(\Delta).
\]

Furthermore

\[
\left\| \int_{\mathbb{R}^s} d^s x \ E(\Delta) \alpha_x (L_0^* L_0) E(\Delta) \right\| \leq N(\Delta, \Gamma) \int_{\mathbb{R}^s} d^s x \ \left\| [\alpha_x (L_0), L_0^*] \right\|
\]

for suitable \( N(\Delta, \Gamma) \in \mathbb{N} \), depending on \( \Delta \) and \( \Gamma \).
Spectral Seminorms

Further Results

Corresponding propositions hold for operators $L$ in the left ideal $\mathcal{L}$ and for detectors $C$ in the algebra $\mathcal{C}$. These results constitute a sharpened version of the initial heuristic idea and allow for the definition of seminorms $q_\Delta$ and $p_\Delta$ on $\mathcal{L}$ and $\mathcal{C}$, respectively.

Definition

$$q_\Delta(L)^2 = \sup_{\omega \in \mathcal{B}(\mathcal{H})^+_{*,1}} \int_{\mathbb{R}^s} d^s x \, \omega(E(\Delta)\alpha_x(L^*L)E(\Delta)),$$

$$p_\Delta(C) = \sup_{\phi \in \mathcal{B}(\mathcal{H})^+_{*,1}} \int_{\mathbb{R}^s} d^s x \, |\phi(E(\Delta)\alpha_x(C)E(\Delta))|. $$
Properties of the Spectral Seminorms

For $A \in \mathcal{A}$, $L \in \mathcal{L}$, $C \in \mathcal{C}$:

- $q_\Delta(AL) \leq \|A\| q_\Delta(L)$, $\|LE(\Delta)\| \leq c(\Delta)q_\Delta(L)$,
- $p_\Delta(L_1^*AL_2) \leq \|E(\Delta_1)AE(\Delta_2)\| q_\Delta(L_1)q_\Delta(L_2)$, $\Delta_i \subseteq \Delta + \Gamma_i$.
- $p_\Delta(L^*L) = q_\Delta(L)^2$, $p_\Delta(C^*) = p_\Delta(C)$.
- Translation invariance:
  $q_\Delta(\alpha_x(L)) = q_\Delta(L)$, $p_\Delta(\alpha_x(C)) = p_\Delta(C)$.
- Continuity of the mapping
  $(\Lambda, x) \mapsto \alpha_{(\Lambda,x)}(L)$.
- Infinite differentiability of the mapping
  $(\Lambda, x) \mapsto \alpha_{(\Lambda,x)}(L_0)$, $L_0 \in \mathcal{L}_0$. 
Linear Functionals on the Algebra of Counters

Functionals Generated by States of Bounded Energy

Let $\omega$ be a physical state with bounded energy, i.e., $\omega(E(\Delta)) = 1$ for suitable $\Delta$, and consider for any $C \in \mathcal{C}$ and continuous function $v \mapsto h(v)$ the functional

$$\rho_{h,t}(C) \equiv T(t)^{-1} \int_t^{t+T(t)} d\tau \int_{\mathbb{R}^s} ds \ x \ h(\tau^{-1}x) \omega(\alpha(\tau,x)(C)).$$

Existence of Limits

*Equicontinuity* of this net of functionals with respect to the seminorm $p_\Delta$ implies, according to the Theorem of Alaoğlu-Bourbaki, the existence of limit functionals on $\mathcal{C}$,

$$\rho_{h,t}(C) \xrightarrow{t \to +\infty} \sigma_{h,\omega}^{(+)}(C),$$

$$\rho_{h,t}(C) \xrightarrow{t \to -\infty} \sigma_{h,\omega}^{(-)}(C).$$
Properties of the Limit Functionals

- \( \sigma \) is a positive linear functional on \( \mathcal{C} = \mathcal{L}^* \cdot \mathcal{L} \).
- Translation invariance: \( \sigma(\alpha_x(C)) = \sigma(C) \).
- Continuity of the mapping 
  \( (\Lambda, x) \mapsto \sigma(L_1^* \alpha_{(\Lambda, x)}(L_2)) \).
- Support of the mapping
  \[
  p \mapsto \int_{\mathbb{R}^{s+1}} d^{s+1}x \exp(-i p^\mu x_\mu) \sigma(L_1^* \alpha_x(L_2))
  \]
  lies in \( \overline{V}_+ - q \) for suitable \( q \in \overline{V}_+ \).
- Cluster property for almost local \( C_1, C_2 \in \mathcal{C} \),
  \[
  \int_{\mathbb{R}^s} d^s x |\sigma(C_1 \alpha_x(C_2))| < \infty.
  \]
Particle Weights

Sesquilinear Forms

Functionals $\sigma$ of the above kind define sesquilinear forms on the left ideal $\mathcal{L}$ with the associated characteristic properties by way of the definition $\langle L_1 | L_2 \rangle_{\sigma} = \sigma(L_1^* L_2)$. These allow for the construction of the associated GNS representations $(\pi_{\sigma}, \mathcal{H}_{\sigma}, U_{\sigma})$ (of the quasilocal algebra $\mathcal{A}$).

Definition of Particle Weights

Sesquilinear forms $\langle \cdot | \cdot \rangle$ on $\mathcal{L}$ complying with all the characteristics given above are called particle weights.

Interpretation

Particle weights represent mixtures of particle-like quantities with sharp energy-momentum (translation invariance), but singly localised at all times (cluster property).
Disintegration Theory

Pure Particle Weights

Elementary systems ought to be represented by pure components associated to irreducible representations of the quasilocal algebra $\mathfrak{A}$ and should be constructed in some way from the highly reducible GNS representation of a generic particle weight.

Warning on Standard Disintegration Theory!!!

The standard disintegration theory presented in the literature on operator algebras is misleading insofar as the authors more often than not fail to mention the indispensable assumption that the algebras to be represented and the representation space have to be norm separable, i.e. contain a countable dense subset. This is NOT the situation that we have at hand; the quasilocal algebra is far more complicated.
Procrustes (and Theseus)

Strategy to overcome the difficulty with separability assumptions:

- Smash the basic structure and what we have carefully developed so far to pieces until only separable constructs are left (localisation regions, net of local algebras, quasilocal algebra, Minkowski space, Poincaré group, ...).

- Use the standard disintegration theory to get the irreducible components.

- Check that upon disintegration what you end up with are still particle weights with all their characteristic features.

- Use the phase space criterion mentioned in the beginning to establish that during this brute-force treatment no information gets lost, i.e. recover the complete structure from the pieces.
The above procedure yields a decomposition in terms of pure particle weights resp. irreducible representations

\[ \langle . | . \rangle_\xi \leftrightarrow (\pi_\xi, \mathcal{H}_\xi) \]

according to

\[ |L\rangle = \int d\mu(\xi) |L\rangle_\xi. \]

Existence of a canonical group of unitaries implementing spacetime translations according to (Borchers and Buchholz 1985) yields

\[ U_\xi(x)|L\rangle_\xi \equiv \exp(i p_\xi^\mu x_\mu)|\alpha_x(L)\rangle_\xi \]

with spectrum in \( \overline{V}_+ \) and uniquely assigned energy-momentum \( p_\xi^\mu \).
Dirac Kets and Lorentz Transformations

**Dirac Kets and Pure Particle Weights**

Dirac's improper energy-momentum eigenstates $|p_0, p\rangle$ correspond to pure particle weights $\langle . | . \rangle_\xi$ with energy-momentum $p_\xi$. In both cases normalisable vectors $L|p_0, p\rangle$ and $|L\rangle_\xi$, respectively, are produced via localisation.

**Lorentz Transformations and Pure Particle Weights**

Lorentz transformations $\Lambda$ leave the class of pure particle weights invariant,

- $\langle . | . \rangle_\xi \mapsto \langle . | . \rangle_\xi^\Lambda \doteq \langle \alpha_{\Lambda^{-1}}( . )|\alpha_{\Lambda^{-1}}( . )\rangle$,
- $\pi_\xi \mapsto \pi_\xi^\Lambda$,
- $p_\xi \mapsto p_\xi^\Lambda \doteq \Lambda p_\xi$. 
An analysis of the joint spectrum of the generators of the intrinsic unitary representation of the translation group, \( x \mapsto U_\xi(x^0, x) \) exhibits that its lower bound has a discrete part at the energy \( E = p_\xi^0 \), which yields the uniform definition of mass for particles as well as infraparticles.

They can be discriminated by the corresponding representations:

- **Particles:** \( \pi_\xi \simeq \pi_\xi^\Lambda \) (unitary equivalence);
- **Infraparticles:** \( \pi_\xi \perp \pi_\xi^\Lambda \) (disjointness).
Spin

Under the assumption of the mass multiplets of the theory to be finite, one gets a continuous representation by matrices of the little group pertaining to the energy-momentum $p_\xi$ and thus an inner projective representation of the little group.

In three space dimensions this means:

1. $p_\xi^2 = m^2 > 0$: little group $SO(3)$, projective representation induced by $SU(2) \implies$ particle weights with definite spin;

2. $p_\xi^2 = 0$: little group $E(2) \implies$ possibility of massless particle weights without definite helicity.
Summary

• The concept of particles is a asymptotic in nature and founded on a suitable notion of locality
• Particle structure is encoded in the geometry of the positive cone of particle weights
• Definition of mass and spin by way of a uniform treatment of particles as well as infraparticles

Tasks

• Spin and statistics and superselection structure associated with particle weights
• Better understanding of disintegration theory, based on the analysis of phase space properties (Choquet theory)
• Particle concept in curved spacetimes?
Collision cross sections in terms of local observables.

The energy-momentum spectrum in local field theories with broken Lorentz-symmetry.

Gauss’ law and the infraparticle problem.

