Strongly Coupled Anisotropic Plasma in AdS/CFT

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Outline

1. Introduction and motivation
2. The theory
3. The Static potential
4. Drag Force
5. The jet Quenching
6. Conclusions
AdS/CFT correspondence

- The AdS/CFT correspondence, in its initial form is a duality between the $\mathcal{N} = 4$ supersymmetric Yang-Mills and type IIB superstring theory on $AdS_5 \times S^5$.

- In this correspondence there exist a map between gauge invariant operators in field theory and states in string theory.

- Example: The Wilson loop, is a physical gauge invariant object and can measure the interaction potential between the external quarks and acts as an order of confinement.

- The Wilson loop operator in the fundamental representation is dual to a string worldsheet extending in the $AdS_5 \times S^5$ with boundary the actual loop placed on the $AdS$ boundary. [Maldacena; Rey, Yee, 1998]

\[
< W[C] > = e^{-S_{string}[C]}
\]
Since the initial correspondence was found, there exist a lot of effort to understand better the gauge/gravity duality.

Another direction is to construct gauge/gravity dualities that can be though as initial models to describe realistic systems and theories.

In our case we are interested for a model that can be used to describe an anisotropic phase of the QGP.
Why do we need anisotropic theories?

- It is interesting theoretically to study thermodynamics and observables in anisotropic IIB SUGRA solutions. The modification of the results due anisotropy and their dependence on it, can be found.
- In early stages the QGP is not in equilibrium. It appears to have anisotropies.
Elliptic flow

Pressure gradients for non-central collisions along the short axis of the elliptic flow are higher than the long axis. Therefore the expansion along the short axis is more rapid leading to anisotropic momentum distribution.

The elliptic flow parameter

\[ v_2 = \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} \]

can be measured experimentally through the particle distributions and through hydrodynamic simulations is predicted low thermalization time.
Longitudinal expansion

- By considering completely central collisions we focus more on the **longitudinal anisotropic expansion**.

- At \( \tau = \tau_0 \) the partonic momentum distributions can be assumed isotropic. Directly after that rapid longitudinal expansion along the beam line which leads to longitudinal cooling and local momentum and pressure anisotropy \( P_L < P_T \).

- Finally the plasma becomes and remains isotropic for \( \tau \geq \tau_{iso} \). After this time hydrodynamics can be applied.

- Lots of work in gauge/gravity duality for \( \tau \geq \tau_{iso} \). Here we study certain quantities in the earlier time where the plasma is anisotropic.
Motivation

- The expansion of the plasma along the longitudinal beam axis at the earliest times after the collision results to momentum anisotropic plasmas.
- Properties of the supergravity solutions, that are dual to the anisotropic plasmas.
- There exist several results for observables in weakly coupled anisotropic plasmas. Do their predictions carry on in the strongly coupled limit models?
- The main question we answer accurately here is: How the inclusion of anisotropy modifies the results on several observables in our dual QGP compared to the isotropic theory?
How does Anisotropy is introduced?

- Introduction of additional branes. [Azeyanagi, Li, Takayanagi, 2009]

<table>
<thead>
<tr>
<th>$x_0$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$u$</th>
<th>$S^5$</th>
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<tbody>
<tr>
<td><strong>D3</strong></td>
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</table>

- Which equivalently leads to the following deformation diagram.

\[
\tau = \frac{\vartheta}{4\pi} + \frac{4\pi i}{g_{\text{ym}}} = \chi + ie^{-\phi}
\]
The anisotropic background

The metric in string frame

\[ ds^2 = \frac{1}{u^2} \left( -FB \, dx_0^2 + dx_1^2 + dx_2^2 + \mathcal{H} \, dx_3^2 + \frac{du^2}{F} \right) + \mathcal{Z} \, d\Omega_5^2. \]

The functions \( F, B, \mathcal{H} \) depend on the radial direction \( u \) and the anisotropy. The anisotropic parameter is \( \alpha \) with units of inverse length. In sufficiently high temperatures, \( T \gg \alpha \), and imposed boundary conditions the Einstein equations can be solved analytically:

\[ F(u) = 1 - \frac{u^4}{u_h^4} + \alpha^2 \frac{1}{24u_h^2} \left[ 8u^2(u_h^2 - u^2) - 10u^4 \log 2 + (3u_h^4 + 7u^4) \log \left( 1 + \frac{u^2}{u_h^2} \right) \right] \]

\[ B(u) = 1 - \alpha^2 \frac{u_h^2}{24} \left[ \frac{10u^2}{u_h^2 + u^2} + \log \left( 1 + \frac{u^2}{u_h^2} \right) \right], \quad \mathcal{H}(u) = \left( 1 + \frac{u^2}{u_h^2} \right)^{\frac{\alpha^2 u_h^2}{4}} \]

The isotropic limit \( \alpha \to 0 \) reproduce the well know result of the isotropic D3-brane solution (dual to \( \mathcal{N} = 4 \) finite sYM solution).
The metric can be expressed in $\alpha, T$ parameters through

$$u_h = \frac{1}{\pi T} + \alpha^2 \frac{5 \log 2 - 2}{48 \pi^3 T^3}.$$  

The pressures can be found from the expectation value of the stress tensor, where the elements $\langle T_{11} \rangle = \langle T_{22} \rangle = P_{x_1 x_2}$ denote the pressure along the $x_1$ and $x_2$ directions and $\langle T_{33} \rangle = P_{x_3}$ is the pressure along the anisotropic direction. The analytic expression read

$$P_{x_1 x_2} = \frac{\pi^2 N_c^2 T^4}{8} + \alpha^2 \frac{N_c^2 T^2}{32}.$$  

$$P_{x_3} = \frac{\pi^2 N_c^2 T^4}{8} - \alpha^2 \frac{N_c^2 T^2}{32}.$$  

$$P_{x_3} < P_{x_1 x_2}$$

resembling the plasma pressure anisotropies.
Target:

- To calculate several observables in the anisotropic theory.
- To study their dependence on the anisotropy.
- To compare the results along the different directions (obtaining dimensionless quantities) and the isotropic theory.
- Static potential, diffusion time, jet quenching are studied.
- Reminder of the notation:
  \[ Q_{\parallel} := Q_{x_3} = Q_{\text{anisotropic}} \]
  \[ Q_{\perp} := Q_{x_1 \text{ or } x_2} \]

  \[ \frac{Q_{\parallel}}{Q_{\perp}} = ?, \quad \frac{Q_{\parallel}}{Q_{\text{iso}}} = ?, \ldots \]
We consider a string world-sheet \((\tau, \sigma)\) of the following form.

\[
\begin{align*}
  x_0 &= \tau, \\
  x_p &= \sigma, \\
  u &= u(\sigma).
\end{align*}
\]

The \(x_p\) is the direction where the pair is aligned:
- \(x_p = x_2 =: x_{\perp}\) pair along transverse direction,
- \(x_p = x_3 =: x_{\parallel}\) pair along parallel direction to anisotropy.

The solution to Nambu-Goto action

\[
S = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{-\tilde{g}}
\]

is a catenary shape w-s with \(u_0\) being the turning point.

We can work in full generality by renaming for example the anisotropic metric as

\[
ds^2 = g_{00}dx_0^2 + \sum g_{ii}dx_i^2 + g_{pp}dx_p^2 + g_{uu}du^2 + \text{internal space}
\]
To find the **static potential** we need to derive from the eoms of the NG action the length $L$ of the Wilson loop. Then express the minimal surface (∼static potential) in terms of $L$. The process is not always doable analytically. In general the length of the two endpoints of the string on the boundary is given by

$$L = 2 \int_{\infty}^{u_0} \frac{du}{u'} = 2 \int_{u_0}^{\infty} du \sqrt{-g_{uu} \frac{c_0^2}{(g_{00} g_{pp} + c_0^2) g_{pp}}}.$$ 

Which should be inverted as $u_0(L)$. The normalized energy of the string is

$$2\pi \alpha' V = c_0 L + 2 \left[ \int_{u_0}^{\infty} du \sqrt{-g_{uu} g_{00}} \left( \sqrt{1 + \frac{c_0^2}{g_{pp} g_{00}}} - 1 \right) - \int_{u_h}^{u_0} du \sqrt{-g_{00} g_{uu}} \right].$$

[Sonnenschein,..]
Therefore we can always at least numerically find the \( V(L) \) expression for any background. In the anisotropic case we get:

- \( V_{\parallel} < V_{\perp} < V_{iso} \) when the comparison is done with \( LT \) keeping \( \alpha, T \) fixed.
- \( \alpha_1 < \alpha_2 \Rightarrow V_{\parallel_1} > V_{\parallel_2} \). Increase of anisotropy, leads to decrease of the static potential.
- The critical length of the string beyond the quarks are not bounded is decreased in presence of anisotropy as \( L_{c\parallel} < L_{c\perp} < L_{c\ iso} \).

![Graph](image.png)

**Figure:** \( V_{\parallel}/V_{\perp}, V_{\parallel}/V_{iso}, V_{\perp}/V_{iso} \) vs \( LT \) and \( T = 3 \), \( \alpha = 0.35T \).
Inclusion of dynamical quarks in isotropic theories leads to screening of the static potential. [Satz; Karsch, Kharzeev, Satz; D.G., Irges, ..]

Inclusion of anisotropy in presence of dynamical quarks does not certainly means further screening. For example in case that the density of dynamical quarks depends on the anisotropy, screening or strengthening may happen.

Weak coupling models have found increased potential in presence anisotropy in the order $V_\parallel > V_\perp > V_{iso}$. Inverse order with our strongly coupled results but still the anisotropic direction is affected mostly. Many differences however between the models. [Dumitru, Guo, Strickland, 2007]
Drag Force

In AdS/CFT the drag force of a single quark moving in the anisotropic plasma can be represented by a trailing string from the boundary where the probe quark moves with the constant speed, to the horizon of the black hole. [Herzog, Karch, Kovtun, Kozcaz, Yaffe; Gubser, 2006]

Again consider the generic metric:

$$ds^2 = g_{00} dx_0^2 + \sum g_{ii} dx_i^2 + g_{pp} dx_p^2 + g_{uu} du^2 + \text{internal space}$$

String Configuration

In radial gauge the trailing string motion along the $x_p := x_{||, \perp}$ directions described by:

$$x_0 = \tau, \quad u = \sigma, \quad x_p = v \tau + \xi(u)$$
Calculating the momentum flowing from the boundary to the bulk we can find the drag force for any background to be

\[ F_d = -\Pi_u^1 = -\sqrt{\lambda} \frac{\sqrt{-g_{00}g_{pp}}}{(2\pi)} \bigg|_{u=u_0} \]

where here \( u_0 \) is given by

\[ (g_{uu}(g_{00} + g_{pp}v^2)) \bigg|_{u=u_0} = 0 \, . \]

The analytic form of the expression in our anisotropic background is found to be:

**Anisotropic Drag Force**

\[
F_{\text{drag}||} = F_{\text{iso}} - \alpha^2 f_1(v), \quad f_1(v) > 0, \quad F_{\text{iso}} < 0
\]

\[
F_{\text{drag}\perp} = F_{\text{iso}} - \alpha^2 f_2(v), \quad f_2(v_c) = 0, \Rightarrow f_2(v) \geq 0
\]
The qualitative behavior is

- $F_{\parallel} > F_{iso}$
- $F_{\perp} > F_{iso}$ for $v > v_c \simeq 0.9$, while below this velocity $F_{\perp} < F_{iso}$.

\[
\frac{F_{\parallel}}{F_{\perp}} = 1 + \alpha^2 \left(2 - v^2\right) \log \left[1 + \sqrt{1 - v^2}\right] \frac{8\pi^2 T^2}{\left(1 - v^2\right)}.
\]

For any velocity: $F_{drag,||} > F_{drag,\perp}$

Figure: $F_{drag,||}/F_{drag,\perp}$, $F_{drag,||}/F_{drag,iso}$, $F_{drag,\perp}/F_{drag,iso}$, vs $\alpha/T$, $v \simeq 0.98$ and $T = 1.$
Diffusion time

Therefore the **diffusion time** $\tau_D$ is given by:

$$\tau_{D,\|,\perp} = \frac{1}{n_{D,\|,\perp}} = -\frac{1}{F_{\text{drag},\|,\perp}} \frac{M_q v}{\sqrt{1 - v^2}},$$

**Remark**

The quark mass receives corrections from the thermal medium. The corrected mass, does not depend on the directions along the probe quark moves in the anisotropic background but depends on the anisotropy.

$$\frac{\tau_{D,\|}}{\tau_{D,\perp}} = 1 - \alpha^2 \frac{(2 - v^2) \log \left[ 1 + \sqrt{1 - v^2} \right]}{8\pi^2 T^2 (1 - v^2)}$$

If we consider sub-leading the thermal mass corrections then the relations between the diffusion times in different directions are inverse to the drag force ones, i.e. $\tau_{D,\|} < \tau_{D,\perp} < \tau_{D,\text{iso}}$. 

In the gravity dual description the jet quenching can be calculated from the minimal surface of a world-sheet which ends on an orthogonal Wilson loop lying along the light-like lines. Two parallel lines of the Wilson loop, with length say $L_-$ related to the partons moving at relativistic velocities are taken to be much more larger that the other two sides with length $L_\perp$ related to the transverse momentum of the radiated gluons.

$$\langle W(C) \rangle = \exp \left( -\frac{1}{4\sqrt{2}} \hat{q} L_\perp^2 L_- \right)$$

[Liu,Rajagopal,Wiedermann,2006]

• The parameter is a measure of energy loss of the quark.
We go to the light-cone coordinates as $\sqrt{2}x^\pm = x_0 \pm x_p$ where $i, p, k = 1, 2, 3$. A generic metric becomes

$$
\begin{align*}
    ds^2 &= g_{--}(dx_+^2 + dx_-^2) + g_{+-}(dx_+ dx_-) + g_{ii(i\neq p)}dx_i^2 + g_{uu}du^2 \\
    g_{--} &= \frac{1}{2}(g_{00} + g_{pp}), \quad g_{+-} = g_{00} - g_{pp}
\end{align*}
$$

String configuration

$$
\begin{align*}
    x_- &= \tau, \quad x_k = \sigma, \quad u = u(\sigma) \\
    x_+, \ x_p\neq k \ &\text{are constant}
\end{align*}
$$

The indices $k, p$ denote a chosen direction.

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<thead>
<tr>
<th>$\hat{q}$</th>
<th>$x_p$</th>
<th>$x_k$</th>
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<th>Momentum broadening along</th>
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<tbody>
<tr>
<td>$\hat{q}_{\perp(\parallel)}$</td>
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<td>$\hat{q}_{\parallel(\perp)}$</td>
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<td>$\hat{q}_{\perp(\perp)}$</td>
<td>$x_{\perp,1}$</td>
<td>$x_{\perp,2}$</td>
<td>$x_{\perp,1}$</td>
<td>$x_{\perp,2}$</td>
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</tbody>
</table>
By calculating the on-shell action, canceling the divergences and applying approximations we obtain

$$\hat{q}_p (k) = \frac{\sqrt{2}}{\pi \alpha'} \left( \int_0^{u_h} \frac{1}{g_{kk} \sqrt{g_{uu}}} \right)^{-1}.$$ 

And • $\hat{q}_{\parallel(\perp)} > \hat{q}_{\perp(\parallel)} > \hat{q}_{\perp(\perp)} > \hat{q}_{iso}$.

$\hat{q}$ (q motion parallel to anisotropy, broadening along transverse) $> \hat{q}$ (q motion transverse to anisotropy, broadening along parallel) $> \hat{q}$ (q motion transverse to anisotropy, broadening along transverse)

![Figure: $\hat{q}_{\perp}$, $\hat{q}_{\perp(\perp)}$ vs $\alpha/T$. $T = 5$.](image-url)
Other Extensions

- Extensions of the jet quenching and the drag force calculations to generic directions and larger anisotropies have been done. [Chernicoff, Fernandez, Mateos, Trancanelli, 2012a,b].
- Results get different for larger anisotropies but also the inequality of pressures does differ.
Anisotropic momentum distribution function in weakly coupled plasmas

The anisotropic distribution function that can be written as

\[ f_{\text{aniso}} = c_{\text{norm}}(\xi)f_{\text{iso}}\left(\sqrt{p^2 + \xi(p \cdot n)^2}\right) \]

where

\[ \xi = \frac{\langle p_T^2 \rangle}{2\langle p_L^2 \rangle} - 1 \]

and \( n \) the unit vector along the anisotropic direction.

[Romatschke, Strickland, 2003]
To relate $\xi$ and $\alpha$ we use the pressures

$$
\Delta := \frac{P_T}{P_L} - 1 = \frac{P_{x_1 x_2}}{P_{x_3}} - 1.
$$

Using the anisotropic distribution function:

$$
\Delta = \frac{1}{2}(\xi - 3) + \xi \left( (1 + \xi) \frac{\arctan \sqrt{\xi}}{\sqrt{\xi}} - 1 \right)^{-1}
$$

Using the supergravity model

$$
\Delta = \frac{\alpha^2}{2\pi^2 T^2}.
$$
For

\[ T \gg \alpha \Rightarrow \xi \ll 1 \Rightarrow \xi \simeq \frac{5\alpha^2}{8\pi^2 T^2}. \]

Supposing we trust the estimation of the anisotropic parameter \( \xi \simeq 1 \) obtained from

\[ \xi = \frac{10\eta}{T \tau s}. \]

and using any comparison normalization scheme (direct or fixed energy or entropy density scheme)

\[ \xi_{aSYM} \gtrsim \xi. \]

In our model \( \xi \ll 1 \) so for \( \xi \simeq 1 \) values that correspond to the QCD anisotropic plasma, our approximations are not valid.
We have calculated several observables using a IIB supergravity solution in the dual anisotropic finite temperature $\mathcal{N} = 4$ sYM plasma.

- The static potential:
  - $V_\parallel < V_\perp < V_{iso}$.
  - $\alpha_1 < \alpha_2 \Rightarrow V_{\parallel 1} > V_{\parallel 2}$.

- The drag Force:
  - $F_\parallel > F_{iso}$ and $F_\parallel > F_\perp$.
  - $F_\perp > F_{iso}$ for $v > v_c \simeq 0.9$, while below this velocity $F_\perp < F_{iso}$.

- The jet quenching:
  - $\hat{q}_{\parallel(\perp)} > \hat{q}_{\perp(\parallel)} > \hat{q}_{\perp(\perp)} > \hat{q}_{iso}$.
  - In weak coupling has been observed enhancement of the jet quenching as $\hat{q}_{\perp(\parallel)} > \hat{q}_{\perp(\perp)}$ in agreement with our results. [Dumitru, Nara, Schenke, Strickland; Baier, Mehtar-Tani, 2008,..].
\[ P(k_\perp) = \int d^2x_\perp e^{-ik_\perp \cdot x_\perp} W_R(x_\perp) \]

\[ W_R(x_\perp) = \frac{1}{d(R)} \langle \text{Tr} \left[ W_R^\dagger[0, x_\perp] W_R[0, 0] \right] \rangle \]

\[ W_R [x^+, x_\perp] \equiv P \left\{ \exp \left[ ig \int_0^{L^-} dx^- A_R^+ (x^+, x^-, x_\perp) \right] \right\} \]

\[ \hat{q} \equiv \frac{\langle k_\perp^2 \rangle}{L} = \frac{1}{L} \int \frac{d^2k_\perp}{(2\pi)^2} k_\perp^2 P(k_\perp) \]