Fast Boundary Element Methods
based on the Adaptive Cross Approximation

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We discuss efficient numerical methods for the boundary integral formulation of various three dimensional boundary value problems for the Laplace equation, Helmholtz equation, system of Lamé equations and system of Maxwell equations for the electromagnetic scattering. The corresponding boundary integral equations will be discretised using Galerkin method leading to a system of linear equations with a dense matrix \( A \) of dimension \( N \). A naive strategy for the solution of the corresponding linear systems would need at least \( O(N^2) \) arithmetical operations and memory. Methods such as fast multipole [1] provide an approximation to the solution vector \( x \) in almost linear complexity. These methods are based on explicitly given kernel approximations by degenerate kernels, i.e. a finite sum of separable functions, which may be seen as a blockwise low-rank approximation of the system matrix. The blockwise approximant permits a fast matrix-vector multiplication, which can be exploited in iterative solvers, and can be stored efficiently. In contrast to the methods mentioned the Adaptive Cross Approximation method (ACA) [2, 3] generates the low-rank approximant from the matrix itself using only few entries and without any explicit a priori known degenerate kernel approximation. The efficiency and convergence properties of the numerical method (Galerkin discretisation, ACA approximation of matrices, iterative solution) will be illustrated for a number of different boundary value problems and for different surfaces [4]. Some numerical examples will be solved in a live computer presentation.

References


