Feb. 2012, Wits, Johannesburg

3-dimensional Born-Infeld Gravity

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Plan

- Motivations
- BI theory and AdS solutions
- AdS wave solution and Log gravity
- Adding ChernSimons term
- Concolusions

♣ *Motivations*

◇ 3D gravity

No local propagating degrees of freedom BUT asymp. AdS_3 solutions.

◇ Deforming the theory

TMG Unitary, parity breaking, a single massive mode of helicity ± 2

[Deser, Jackiw, Templeton(82)]

$$I_{TMG} = \frac{1}{2\kappa^2} \int d^3x \sqrt{-g} \left[R - 2\Lambda + \frac{1}{\mu} \epsilon^{\lambda\mu\nu} \left(\Gamma^{\rho}{}_{\lambda\sigma} \partial_\mu \Gamma^{\sigma}{}_{\rho\nu} + \frac{2}{3} \Gamma^{\rho}{}_{\lambda\sigma} \Gamma^{\sigma}{}_{\mu\tau} \Gamma^{\tau}{}_{\nu\rho} \right) \right]$$

NMG Unitary, parity preserving, graviton with 2 polarization states of helicity ± 2

[Bergshoeff, Hohm, Townsend(09)]

$$I_{NMG} = \frac{1}{2\kappa^2} \int d^3x \sqrt{-g} \left[R - 2\Lambda + \frac{1}{m^2} (R_{\mu\nu} R^{\mu\nu} - \frac{3}{8} R^2) \right]$$

◇ BI-NMG Generalizing and keeping the unitarity

[Gullu, Sisman, Tekin(10)]

$$I = -\frac{4m^2}{\kappa^2} \int d^3x \sqrt{-\det g} \left[\sqrt{-\det\left(1 - \frac{1}{m^2} g^{-1} G\right)} - \left(1 + \frac{\Lambda}{2m^2}\right) \right]$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}, \qquad \kappa^2 = 16\pi G_3$$

Interesting points:

At the 1st order $\rightarrow$ Einstein-Hilbert action + cosmological constant

At the 2nd order $\rightarrow$ NMG action

A unique vacuum solution (degeneracy in finite order deformations)

♣ BI theory and AdS solutions

Equation of motion:

$$
\begin{aligned}
0 = & -\frac{1}{2} F g_{\mu\nu} + \left(g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu \right) F_R + F_R R_{\mu\nu} \\
& + \frac{1}{m^2} \Big\{ 2 \nabla_\alpha \nabla_\mu \left(F_R R^\alpha{}_\nu \right) - g_{\mu\nu} \nabla_\beta \nabla_\alpha \left(F_R R^{\alpha\beta} \right) - \Box \left(F_R R_{\mu\nu} \right) - 2 F_R R_\nu{}^\alpha R_{\mu\alpha} \\
& \qquad + g_{\mu\nu} \Box \left(F_R R \right) - \nabla_\mu \nabla_\nu \left(F_R R \right) + F_R R R_{\mu\nu} \Big\} \\
& - \frac{1}{2m^4} \Big\{ 4 F_R R^\rho{}_\mu R_{\rho\alpha} R^\alpha{}_\nu + 2 g_{\mu\nu} \nabla_\alpha \nabla_\beta \left(F_R R^{\beta\rho} R^\alpha{}_\rho \right) + 2 \Box \left(F_R R_\nu{}^\rho R_{\mu\rho} \right) \\
& \qquad - 4 \nabla_\alpha \nabla_\mu \left(F_R R_\nu{}^\rho R^\alpha{}_\rho \right) + 2 \nabla_\alpha \nabla_\mu \left(F_R R R^\alpha{}_\nu \right) - g_{\mu\nu} \nabla_\alpha \nabla_\beta \left(F_R R R^{\alpha\beta} \right) \\
& \qquad - \Box \left(F_R R R_{\mu\nu} \right) - 2 F_R R R_\nu{}^\rho R_{\mu\rho} - g_{\mu\nu} \Box \left(F_R R^2_{\alpha\beta} \right) + \nabla_\nu \nabla_\mu \left(F_R R^2_{\alpha\beta} \right) \\
& \qquad - F_R R^2_{\alpha\beta} R_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \Box \left(F_R R^2 \right) - \frac{1}{2} \nabla_\mu \nabla_\nu \left(F_R R^2 \right) + \frac{1}{2} F_R R^2 R_{\mu\nu} \Big\}
\end{aligned}
$$

where

$$
F = \left[\sqrt{- \det \left(1 - \frac{1}{m^2} g^{-1} G \right)} - \left(1 + \frac{\Lambda}{2m^2} \right) \right], \qquad F_R = \frac{\partial F}{\partial R}, \qquad \Box = \partial_\mu \partial^\mu
$$

Interesting solutions with $AdS_2 \times S^1$ symmetry $\Rightarrow$ Entropy function

[Sen(05)]

Steps:

1- Choosing an ansatz

$$ds^2 = v_1 \left(-r^2 dt^2 + \frac{dr^2}{r^2} \right) + v_2 (dz + e r dt)^2$$

2- Integrating out the angular coordinate at the ansatz $f = \int dz \sqrt{-g} \mathcal{L}|_{ansatz}$ to find the entropy function

$$S(v_1, v_2, e) = 2\pi (eq - f)$$

3- Extremaizing the Entropy function with respect to v_1, v_2, e.

Advantage

You have the solution AND the entropy of the solution.

Euqations

$$E_1 = \left(-3{v_2}^2e^4 + 3v_1v_2e^2 - 2v_2{v_1}^2e^2m^2 + 4{v_1}^3m^2 - 8\,{v_1}^4m^4\right) + 2\,m\,{v_1}^3\sqrt{\left(3v_2e^2 - 4v_1 + 4\,m^2{v_1}^2\right)}(\Lambda + 2\,m^2) = 0,$$

$$E_2 = \left(3{v_2}^2e^4 - 3v_1v_2e^2 - 3v_2{v_1}^2e^2m^2 + 4{v_1}^3m^2 - 4\,{v_1}^4m^4\right) + m\,{v_1}^3\sqrt{\left(3v_2e^2 - 4v_1 + 4\,m^2{v_1}^2\right)}(\Lambda + 2\,m^2) = 0,$$

$$q = \frac{\pi e {v_2}^{3/2}}{\kappa^2{v_1}^2m}\,\frac{\left(9v_2e^2 - 8v_1 - 4m^2{v_1}^2\right)}{\sqrt{\left(3v_2e^2 - 4v_1 + 4\,m^2{v_1}^2\right)}}$$

Note that

$$E_1 - 2E_2 = (v_2e^2 - v_1)(9v_2e^2 - 4v_1^2m^2) = 0$$

Leads to TWO different solutions:

$$1)\ v_2 = \frac{v_1}{e^2}, \qquad 2)\ v_2 = \frac{4m^2{v_1}^2}{9e^2}$$

1st solution

$$v_1 = -\frac{m^2}{\Lambda(4m^2+\Lambda)}, \qquad e^2 = \frac{\pi}{\kappa^2\, q\, m}\frac{\Lambda + 2\, m^2}{\sqrt{-\Lambda(4\, m^2+\Lambda)}},$$

$$v_2 = \frac{v_1}{e^2} \qquad S = \frac{4\,\pi^2}{\kappa^2\, m\, e}\frac{\Lambda + 2\, m^2}{\sqrt{-\Lambda\left(4\, m^2+\Lambda\right)}}$$

Note that:

1- The entropy is physically meaningful for $\Lambda + 2\, m^2 > 0$.

2- The solution is locally AdS_3 with radius $l^2 = 4v_1 = -\frac{4m^2}{\Lambda(4m^2+\Lambda)}$ or

$$\Lambda = -2m^2\left(1 - \sqrt{1 - \frac{1}{m^2 l^2}}\right).$$

3- Real cosmological constant $\Rightarrow$ $m^2 l^2 \geq 1$.

Microscopic entropy:

The c-extremization approach

[Kraus, Larsen(05)]

$$c = \frac{3l}{2G}\sqrt{1 - \frac{1}{m^2 l^2}}$$

AND

The period of the compact direction $\Rightarrow$ The associated temperature

$$T = \frac{1}{2\pi e} = \frac{2}{\pi}\sqrt{\frac{Gq}{l\sqrt{1-\frac{1}{m^2 l^2}}}}$$

leads to the microscopic entropy

$$S_{mic} = \frac{\pi^2}{3} T c = S_{mac}$$

2nd solution:

$$v_1 = \frac{9}{8}\,\frac{188m^4 - 324\Lambda m^2 - 81\Lambda^2 \pm 9\sqrt{\left(9\Lambda + 34m^2\right)\left(9\Lambda + 2m^2\right)}\left(\Lambda + 2m^2\right)}{m^2\left(52m^4 - 972\Lambda m^2 - 243\Lambda^2\right)},$$

$$v_2 = \frac{4m^2 v_1^2}{9e^2}, \qquad e^2 = -\frac{32\pi}{27\kappa^2 q}\frac{m^2{v_1}^2}{\sqrt{\frac{4}{3}{v_1}^2m^2 - v_1}}$$

$$S = \frac{16\pi^2 m\,{v_1}^2}{81\,\kappa^2\,e\sqrt{\frac{4}{3}{v_1}^2m^2 - v_1}}\left[64\,m^3 v_1 - 60\,m - 27\,(\Lambda + 2\,m^2)\sqrt{\frac{4}{3}{v_1}^2m^2 - v_1}\right]$$

Notes:

1- It is meaningful if $9\Lambda + 2m^2 > 0$.

2- May be interpreted as a warped blak hole solution.

♣ *AdS wave solution and Log gravity*

AdS waves are exact gravitational waves propagating along AdS space.

For TMG and NMG:
At the critical value of the parameters $\rightarrow$ logarithmic behaviors.

[Ayon-Beato, Giribet, Hassaine(05, 06, 09)]

Suitable parametrization for AdS_3 solution:

$$ds^2 = \frac{l^2}{y^2}\left(dy^2 - 2dudv\right), \quad \text{with} \quad l^2 = -\frac{4m^2}{\Lambda(\Lambda + 4m^2)}$$

The ansatz for AdS wave solution:

$$ds^2 = \frac{L^2}{y^2}\left[dy^2 - 2dvdu - G(u,y)du^2\right]$$

where

L: the typical radius (eom $\Rightarrow L = l$)

$G(u,y)$: an arbitrary function (to be determined by eom)

eom leads to

$$\frac{y^4\frac{\partial^4 G}{\partial y^4} + 2\,y^3\frac{\partial^3 G}{\partial y^3} - \,m^2\,l^2\,\left(y^2\frac{\partial^2 G}{\partial y^2} - y\frac{\partial G}{\partial y}\right)}{y^2\,l\,m\,\sqrt{m^2\,l^2 - 1}} = 0$$

The dominator plays a crucial rule.

At the critical point the $c = 0 \Rightarrow m^2 l^2 = 1$.
Moreover: reduction to NMG model in $ml \gg 1$:

$$\frac{1}{y^2\,m^2 l^2}\left[y^4\frac{\partial^4 G}{\partial y^4} + 2\,y^3\frac{\partial^3 G}{\partial y} - \,\frac{2m^2\,l^2 + 1}{2}\,\left(y^2\frac{\partial^2 G}{\partial y^2} - y\frac{\partial G}{\partial y}\right)\right] + \mathcal{O}(\frac{1}{m^4 l^4}) = 0,$$

there are some contributions from dominator.

Standard solution is : $\quad G = y^\alpha$
where α satisfies $\quad \alpha(\alpha - 2)\left[(\alpha - 1)^2 - \,m^2 l^2\right] = 0.$
The generic solution:

$$G(u,y) = \underbrace{G_0(u) + G_2(u)y^2}_{\text{can be eliminated}} + G_+(u)\left(\frac{y}{l}\right)^{1+m\,l} + G_-(u)\left(\frac{y}{l}\right)^{1-m\,l}$$

At $m^2 l^2 = 1$: multiplicity in the roots of the characterestic eq.
BUT the dominator of the eom is zero at this point.
Nevertheless we can look for a possible limiting solution where $m^2 l^2 \to 1$.

Using the ansatz:

$$G(u,y) = \ln\left(\frac{y}{l}\right)\left[G_1(u)\left(\frac{y}{l}\right)^2 + G_2(u)\right],$$

and plugging it into eom $\to$

$$\left.\frac{2(G_1 l^2 - G_2 y^2)}{y^2\, l^3\, m}\sqrt{-1 + m^2 l^2}\right|_{m^2 l^2 \to 1} = 0$$

Although the limit is well defined, the log solution is NOT a solution and it can be treated as a limiting solution. (In contrat with TMG and NMG)

♣ *Adding chern-Simons term*

3D Chern-Simons term:

$$I_{CS} = \frac{1}{2\kappa^2\mu}\int d^3x\,\sqrt{-g}\epsilon^{\lambda\mu\nu}\left(\Gamma^{\rho}{}_{\lambda\sigma}\partial_\mu\Gamma^{\sigma}{}_{\rho\nu} + \frac{2}{3}\Gamma^{\rho}{}_{\lambda\sigma}\Gamma^{\sigma}{}_{\mu\tau}\Gamma^{\tau}{}_{\nu\rho}\right).$$

Interesting: The eom is not corrected by CS term for AdS_3 ansatz.

BUT Parity violating $\Rightarrow c_L \neq c_R$.

c-extremization formalism $\Rightarrow$ $\frac{c_L+c_R}{2} = \frac{3l}{2G}\sqrt{1-\frac{1}{m^2 l^2}}$

[Kraus, Larsen (05)]

Diffeomorphism anomaly $\Rightarrow$ $c_L - c_R = -\frac{3}{G\mu}$

[Kraus, Larsen (06)]

Therefore

$$c_L = \frac{3l}{2G}\left(\sqrt{1-\frac{1}{m^2 l^2}} - \frac{1}{\mu l}\right), \qquad c_R = \frac{3l}{2G}\left(\sqrt{1-\frac{1}{m^2 l^2}} + \frac{1}{\mu l}\right).$$

◇ Chiral model

Is there any point in the moduli space of the parameters where the theory could be chiral?!

[Li, Song, Strominger (08)]

One possibility is $c_L = 0 \Rightarrow$ chiral line:

$$\sqrt{1 - \frac{1}{m^2 l^2}} = \frac{1}{\mu l}.$$

◇ AdS wave solution

Using the same ansatz for AdS wave the eoms leads to

$$\frac{y^4 \frac{\partial^4 G}{\partial y^4} + \left(2 - \frac{m}{\mu}\sqrt{m^2 l^2 - 1}\right) y^3 \frac{\partial^3 G}{\partial y} - m^2 l^2 \left(y^2 \frac{\partial^2 G}{\partial y^2} - y\frac{\partial G}{\partial y}\right)}{y^2 \, l \, m \sqrt{m^2 l^2 - 1}} = 0.$$

which reduces to generalaized massive gravity

[Ayon-Beato, Giribet, Hassaine (09)]

$$\frac{1}{y^2 m^2 l^2}\left[y^4 \frac{\partial^4 G}{\partial y^4} + (2 - \frac{l m^2}{\mu}) y^3 \frac{\partial^3 G}{\partial y} - \frac{2m^2 l^2 + 1}{2}\left(y^2 \frac{\partial^2 G}{\partial y^2} - y \frac{\partial G}{\partial y}\right)\right] + \mathcal{O}(\frac{1}{m^4 l^4}) = 0$$

for $ml \gg 1$. (Crucial rule of the dominator again.)

The characterestic eq for generic solution $G = y^{\alpha}$ is

$$\alpha(\alpha-2)\left[(\alpha-1)^2-(\alpha-1)\frac{m}{\mu}\sqrt{m^2\,l^2-1}-m^2\,l^2\right]=0.$$

The general solution:

$$\begin{aligned} G(u,y) &= G_{+}(u)\left(\frac{y}{l}\right)^{\frac{1}{2\mu}\left(2\mu+m\sqrt{m^2l^2-1}+\sqrt{m^2(m^2l^2-1)+4\mu^2m^2l^2}\right)} \\ &+ G_{-}(u)\left(\frac{y}{l}\right)^{\frac{1}{2\mu}\left(2\mu+m\sqrt{m^2l^2-1}-\sqrt{m^2(m^2l^2-1)+4\mu^2m^2l^2}\right)}. \end{aligned}$$

On the chiral line ($c_L = 0$) $\rightarrow$ multiplicity in characterestic eq

$\Rightarrow$ log solution

$$G(u,y)=G_1(u)\ln\left(\frac{y}{l}\right)+G_2(u)\left(\frac{y}{l}\right)^{m^2l^2+1}.$$

Points

1-At $m^2l^2 = 1$ there is another degeneracy in characterestic eq BUT we are not allowed to set that.

2-The limit of $m^2l^2 \rightarrow 1$ is not well defined and it's not chiral point.

♣ *Conclusions*

◇ Truncating the infinite sum crucially changes the physical content of the theory. One might suspect that the effect of the infinite summation is just shifting the value of the critical point ($c = 0$) but, this is not the case. Summing up the infinite terms resolves the critical point.

◇ The critical point ($c = 0$) is a singular point in the moduli space of the parameters and the point can only be reached in $m^2l^2 \to 1$. The log gravity is a limiting (not exact) solution of BI theory. It would be interesting to explore the meaning of this effect from dual CFT description.

◇ The situation changes drastically by adding CS terms to the BI gravity. Although the $m^2l^2 = 1$ is still a singular point in the moduli space of the parameters, there is a critical chiral line ($c_L = 0$) and the solution develops a logarithmic term. One would expect that the theory along this critical line is dual to a LCFT (as TMG and NMG). It would also be interesting to understand this correspondence better.

◇ It is possible to relax $m > 0, \Lambda < 0$ which leads to other critical points are possible.

◇ Investigating the gravitons could be another direction for future studies.

Thank you for your attention